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Stellar Collisions in Young Clusters: Formation of (Very) Massive Stars?

Marc Freitag

Institute of Astronomy, Madingley road, Cambridge CB3 0HA, UK

Abstract. In young star clusters, the density can be high enough and the velocity dispersion low enough for stars to collide and merge with a significant probability. This has been suggested as a possible way to build up the high-mass portion of the stellar mass function and as a mechanism leading to the formation of one or two very massive stars ($M_* > 150\,\mathrm{M}_\odot$) through a collisional runaway. I quickly review the standard theory of stellar collisions, covering both the stellar dynamics of dense clusters and the hydrodynamics of encounters between stars. The conditions for collisions to take place at a significant rate are relatively well understood for idealised spherical cluster models without initial mass segregation, devoid of gas and composed of main-sequence (MS) stars. In this simplified situation, 2-body relaxation drives core collapse through mass segregation and a collisional phase ensues if the core collapse time is shorter than the MS lifetime of the most massive stars initially present. The outcome of this phase is still highly uncertain. A more realistic situation is that of a cluster still containing large amounts of interstellar gas from which stars are accreting. As stellar masses increase, the central regions of the cluster contracts. This little-explored mechanism can potentially lead to very high stellar densities but it is likely that, except for very rich systems, the contraction is halted by few-body interactions before collisions set in. A complete picture, combining both scenarios, will need to address many uncertainties, including the role of cluster sub-structure, the dynamical effect of interstellar gas, non-MS stars and the structure and evolution of merged stars.

1. Collision and Interaction Timescales

We first consider the timescale for direct collisions between stars. We assume two populations of stars, with stellar masses m_1 and m_2, stellar radii r_1 and r_2, and Maxwellian velocity distributions of (1D) dispersions σ_1 and σ_2. The mean time for a star of type 1 to collide with a star of type 2 is

$$t_{\mathrm{coll}} = \left\{ \sqrt{8\pi} n_2 \sigma_{\mathrm{rel}} (r_1 + r_2)^2 \left[1 + \frac{G(m_1 + m_2)}{\sigma_{\mathrm{rel}}^2 (r_1 + r_2)} \right] \right\}^{-1}, \tag{1}$$

with $\sigma_{\mathrm{rel}}^2 = \sigma_1^2 + \sigma_2^2$ (Binney & Tremaine 1987). The second term in the square brackets originates in the mutual gravitational attraction of the stars. This gravitational focusing dominates over the purely geometrical cross section (represented by the term 1 in brackets) unless the velocity dispersion is similar or higher than the escape velocity from the surface of a star, i.e. $500 - 1000\,\mathrm{km\,s^{-1}}$. Such relative velocities are presumably only reached in the vicinity of massive

black holes (e.g., Freitag & Benz 2002, 2005). For other situations, the collision time can be written

$$t_{\rm coll} \simeq 5\,{\rm Gyr}\,\frac{10^6{\rm pc}^{-3}}{n_*}\frac{\sigma_{\rm rel}}{10\,{\rm km\,s}^{-1}}\frac{2\,{\rm R}_\odot}{r_1+r_2}\frac{2\,{\rm M}_\odot}{m_1+m_2}. \quad (2)$$

In Fig. 1, I show (with dashed lines) the stellar densities required for a massive star to experience on average one collision during the MS. Typical values are in excess of $10^7\,{\rm pc}^{-3}$ or $10^6\,{\rm pc}^{-3}$ for a star of $10\,{\rm M}_\odot$ or $120\,{\rm M}_\odot$, respectively.

Stars passing each other within a few stellar radii at low velocities can dissipate enough orbital energy in tides to form a bound binary (e.g., Fabian, Pringle, & Rees 1975; Kim & Lee 1999). Such a "tidal binary", being born very tight, is likely to merge quickly, or, at least, to enter a common-envelope phase (Dale & Davies 2006). The merger can also be induced by the perturbation of a passing star (Bonnell & Bate 2005). In any case, because tidal dissipation decreases steeply with increasing closest-approach distance, formation and merging of tidal binary can reduce the effective value of the collision time by a factor $\sim 3-5$ at most.

A more promising way to get interesting collision rates is to consider binary interactions. The timescale over which a binary with separation a and mass $m_{\rm bin}$ suffers from a close interaction with a field star is simply

$$t_{\rm bin,inter} \simeq 30\,{\rm Myr}\,\frac{10^6{\rm pc}^{-3}}{n_*}\frac{\sigma_{\rm rel}}{10\,{\rm km\,s}^{-1}}\frac{1\,{\rm AU}}{a}\frac{3\,{\rm M}_\odot}{m_{\rm bin}+m}, \quad (3)$$

where m is the individual mass of the field stars and n_* their number density. For a sufficiently hard binary, i.e. when the total orbital energy of the binary and impactor system is negative, most close interactions are "resonant". This means that the three stars orbit each other in a chaotic fashion for several dynamical times within a volume of order a^3 until this meta-stable system decays into a binary and single star if stars were point masses[1] (e.g., Heggie & Hut 2003). There is in fact a high probability that two of the stars collide and merge during a resonant interaction. For instance, Fregeau et al. (2004) showed that, in the hard regime, the cross section for a collision to occur during an encounter of a circular, $a = 1\,{\rm AU}$ binary consisting of two $1\,{\rm M}_\odot$ MS stars with a similar star is about $\frac{3}{2}\pi a G {\rm M}_\odot v_{\rm rel}^{-2}$, corresponding to a hard sphere of radius $a/4$. If massive binaries are born with eccentricities ≈ 0.9, as a result of accretion of interstellar gas, more distant, non-resonant, interactions with passing stars can suffice to cause a merger, as suggested by Bonnell & Bate (2005).

At relative velocities below about $5\,{\rm km\,s}^{-1}$, encounters involving two massive stars, one or both of them surrounded by a massive disc, can result into a bound binary if the closest approach distance d is smaller than a few tenth of the disc radius, which corresponds typically to $d \approx 10-100\,{\rm AU}$ (Moeckel & Bally 2007b). With d substituted for a, Eq. 3 suggests that this mechanism might be responsible for the high proportion of massive stars with a companion of similar mass in clusters such as the ONC, despite the short life time of massive discs (Moeckel & Bally 2007a; Pfalzner & Olczak 2007).

[1] Binary-binary interactions occur in a similar fashion.

2. The Outcome of Stellar Collisions

Collisions between single MS stars have been the object of extensive numerical studies, starting with the 2–D grid simulations of Seidl & Cameron (1972), with most works based on the Smoothed Particle Hydrodynamics (SPH) method (e.g., Freitag & Benz 2005; see the contributions in Shara 2002 and the "MODEST" web pages[2] for more references on stellar collisions). At low velocities, typical of globular clusters or young clusters, any collision between MS stars lead to a merger with very little mass loss ($< 10\,\%$, typically a few %). Neglecting the possibility of the formation of tidal binaries, stars can therefore be treated as sticky spheres to good approximation.

In this velocity regime, one can determine how the mass elements of the parent stars sort themselves in the collision product, using a algorithm based on Archimedes' principle, to determine the internal structure of the merger product with good accuracy, avoiding the computational cost of an hydrodynamical simulation (Lombardi et al. 2002; Gaburov et al. 2007b). In case of collisions between mass of unequal masses, most of the material from the lower object sinks to the centre of the merged star.

One major remaining uncertainty is the evolution of collision products (Sills et al. 1997, 2001, 2002; Glebbeek & Pols 2007). When the merger has settled down to a new hydrodynamical equilibrium, it is significantly swollen compared to the size of a MS star of the same mass, and typically spinning at a rate close to breakup. Some efficient mechanism, such as winds or magnetic locking to a disc, has to operate to shed most of this angular momentum but comparatively little mass as the star contracts back to the MS, lest it be nearly completely ground down by "centrifugal evaporation" (Sills, Adams, & Davies 2005).

From consideration on cross section and the fact that most massive MS stars are members of binaries (or higher-order systems), it seems that most collisions in a young cluster occur during binary interactions. Unfortunately, because of the added numerical complexity and the enormous parameter space to be explored, only very few researchers have carried out hydrodynamical simulations of such interactions (Goodman & Hernquist 1991; Lombardi et al. 2003). These works made it clear that *multiple* mergers are relatively likely during a binary interactions due to the increased size of collision products (see also Fregeau et al. 2004).

In a very young cluster, one has to consider collisions involving stars still on the pre-MS. Low-mass stars ($m < 5 - 10\,\mathrm{M}_\odot$) take much more time than massive ones to contract to the MS and can therefore be significantly larger that massive stars (e.g., Palla 2002). Only few SPH simulations involving pre-MS stars have been published so far (Zinnecker & Bate 2002; Laycock & Sills 2005; Davies et al. 2006). They show how, in a close encounter with a massive star, a low-mass pre-MS star is tidally disrupted and end up forming a disc around the massive star. As mentioned above, such massive discs offer a very large cross section for the formation of tight binaries which may later merge, a sequence of events investigated by Davies et al. (2006).

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[2] "Modelling Dense Stellar Systems"; see `http://www.manybody.org/modest/`. Follow the link "WG4" for the pages on collisions.

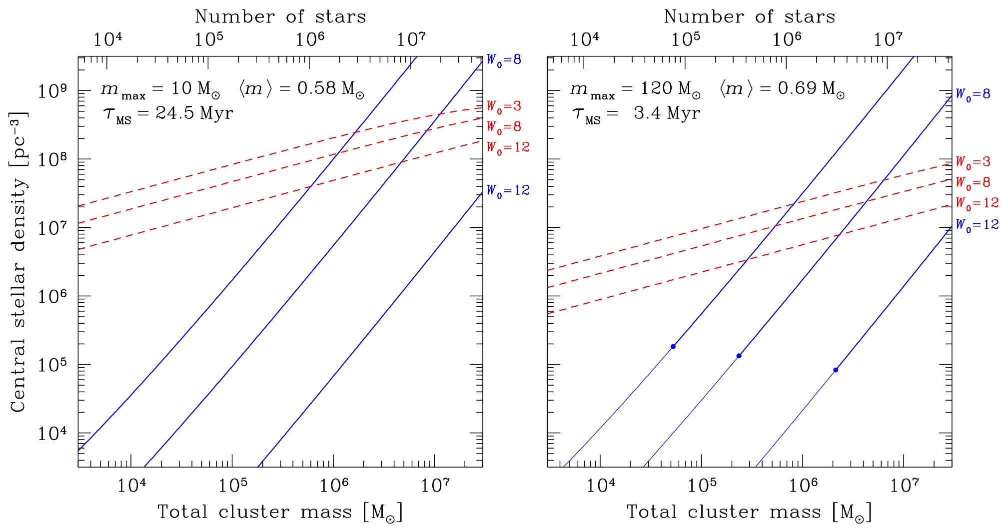

Figure 1. Stellar densities required for high collision rate. The dashed lines show the density at which the collision time $t_{\rm coll}$ for a star of mass $m_{\rm max} = 10\,{\rm M}_\odot$ (left panel) or $120\,{\rm M}_\odot$ (right panel) is equal to its MS lifetime $\tau_{\rm MS}$, at the centre of a non-evolving King-model cluster with $W_0 = 3, 8, 12$ (from top to bottom). The dependence on cluster mass (and on W_0) comes from the dependence of $t_{\rm coll}$ on the velocity dispersion. The solid lines show the central density corresponding to a core-collapse time $t_{\rm cc}$ equal to $\tau_{\rm MS}$. To estimate $t_{\rm cc}$, we assume a Salpeter mass function from $0.2\,{\rm M}_\odot$ to $m_{\rm max}$. The dots on the solid lines (right panel) are a rough estimate of the number of stars above which our estimate of $t_{\rm cc}$ is robust.

3. First Scenario: Relaxation-Driven Core Collapse

Several authors have studied how 2-body relaxation can drive the evolution of a cluster to a stage of very high central density, leading to stellar collisions (Portegies Zwart et al. 1999; Portegies Zwart & McMillan 2002; Portegies Zwart et al. 2004; Gürkan, Freitag, & Rasio 2004; Freitag, Rasio, & Baumgardt 2006b; Freitag, Gürkan, & Rasio 2006a; Gürkan, Fregeau, & Rasio 2006; Gaburov, Gualandris, & Portegies Zwart 2007a). Two-body relaxation is the process through which stars in a populous cluster ($N \gg 10$) exchange energy and angular momentum over a time scale significantly longer than their orbital time, by deflecting each other's trajectory[3]. Roughly speaking, the relaxation time is the timescale for orbital parameters of a an "average star" to change completely as a result of a very large number of uncorrelated small-angle hyperbolic 2-body encounters with all other stars (e.g., Binney & Tremaine 1987). Its local value is given by

$$t_{\rm rlx} \simeq \frac{0.339}{\Lambda}\frac{\sigma^3}{G^2\langle m\rangle^2 n_*} \simeq 2\,{\rm Myr}\,\frac{10}{\Lambda}\frac{10^6{\rm pc}^{-3}}{n_*}\left(\frac{\sigma}{10\,{\rm km\,s^{-1}}}\right)^3\left(\frac{1\,{\rm M}_\odot}{\langle m\rangle}\right)^2, \quad (4)$$

[3] In a smaller group, 2-body relaxation cannot be defined in a useful way because the energy and angular momentum of a star are generally not conserved on a dynamical time.

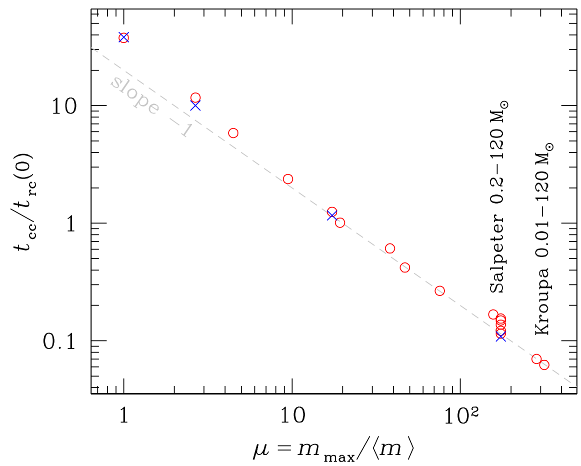

Figure 2. Core collapse times for star clusters with a variety of mass functions. Shown are previously unpublished results of Monte-Carlo simulations by the author (circles) and of N-body simulations by H. Baumgardt (crosses). In most cases, a Plummer model with a MF $dN/dm \propto m^{-2.35}$ was used.

where $\Lambda \simeq \ln(0.02N)$, N is the number of stars in the cluster, σ the 1D velocity dispersion and $\langle m \rangle$ the mean stellar mass.

In a cluster with a mass function (MF), the massive stars lose kinetic energy in favour of lower-mass objects through 2-body relaxation. This would eventually lead to energy equipartition if it was not for the cluster's self-gravity. As they lose energy, the most massive stars concentrate to the centre, a process known as dynamical mass segregation and which reduces to dynamical friction in the limit of very large mass ratio. For any reasonable MF the central subsystem of massive stars becomes self-gravitating before equipartition is established. This makes thermal equilibrium impossible to achieve because the massive subsystem gets denser and hotter at an accelerated rate as it gives up energy to the rest of the cluster, an instability first predicted by Spitzer (1969). Using the Monte-Carlo stellar dynamical method invented by Hénon (1971a,b), Gürkan et al. (2004) showed that in rich clusters this mass-segregation-induced core collapse occurs on a timescale $t_{\rm cc}$ dictated by the initial *central* relaxation time $t_{\rm rc}(0)$, largely independently of the cluster structure,

$$t_{\rm cc} \approx 2\, t_{\rm rc}(0) \frac{10}{\mu} \quad \text{for} \quad \mu \equiv \frac{m_{\rm max}}{\langle m \rangle} > 5. \tag{5}$$

This relation is shown in Fig. 2 for a set of cluster simulations.

The solid lines in Fig. 1 show what is the minimum initial central density of a cluster of given mass and concentration for the core collapse time to be shorter than the MS lifetime $\tau_{\rm MS}$ of the most massive stars. In Monte-Carlo simulations where this is the case, the core collapse proceeds until the massive stars in the

central region start colliding, either as single stars if there are no primordial binaries (a rather unrealistic idealisation) or during binary interactions. When $t_{\rm cc} > \tau_{\rm MS}$, the gas lost by massive star in the post-MS evolution causes the core of the cluster to re-expand before any significant number of collisions can occur.

Simulations carried out with the more accurate but very time-consuming direct N-body algorithm in the regime $\mu \geq 40$ and $N <$ few $\times\, 10^5$ indicate a more constraining condition for a collisional phase to occur (e.g., Portegies Zwart et al. 2004). This is probably because the type of core collapse described here can only occur if there are initially a sufficient number of stars with masses close to $m_{\rm max}$ in a region not much larger than the core. In Fig. 1 we indicate with dots on the solid lines the cluster masses below which there are on average fewer than 5 stars with a mass between $0.5\, m_{\rm max}$ and $m_{\rm max}$ in the core initially. This is not a significant limitation for $m_{\rm max} = 10\, {\rm M}_\odot$.

The outcome of the collisional phase is still a subject of debate. Dynamical simulations which only take into account collisional mass loss and assume that the merger product returns to the MS immediately after a collision show a runaway growth of one or two stars to masses $m_{\rm ra}$ of a few hundreds or thousands ${\rm M}_\odot$. Such "very massive stars" (VMSs) have been suggested as progenitors of intermediate-mass black holes but it seems that a VMS with non-negligible metallicity, if left to evolve on its own, will lose most of its mass by stellar winds and produce only a rather low-mass remnant (Belkus et al. 2007; Yungelson et al. 2007). The collisional growth can also be self-limited if the average time between two mergers becomes shorter than the thermal timescale of the VMS, as seen in Monte-Carlo simulations of very rich cluster of single stars (Freitag et al. 2006a). Then the VMS can not contract back to the MS and might instead become too diffuse to stop impactors.

It is in any case unlikely that this scenario can lead to the formation of a continuous spectrum of massive stars. Simulations as well as simple mathematical models suggest that one (possibly two) VMS detaches itself from the initial MF with very few, if any, stars populating the gap between $m_{\rm max}$ and $m_{\rm ra}$.

4. Second Scenario: Accretion-Driven Core Collapse

The relaxation-driven scenario is based on a simplistic model of young clusters, neglecting the role of non-spherical substructure, initial mass segregation and interstellar gas. The last point is of particular importance because during the first Myr or so of their lives, most of the mass of clusters is in gas rather than stars. When the residual gas is expelled through ionisation by the new born massive stars or, for the most massive clusters, by the first SN explosion, the cluster reacts by expanding considerably, probably to the point of complete dispersion for most low-mass clusters (Geyer & Burkert 2001; Kroupa, Aarseth, & Hurley 2001; Goodwin & Bastian 2006; Baumgardt & Kroupa 2007). If a cluster which remains bound is a factor X_R larger and a factor X_M less massive than in the embedded phase, with $X_R \approx 10$ and $X_N \approx 0.3$ being reasonable values, one can estimate that its relaxation time has *increased* by a factor $X_{\rm rlx} \approx X_R^{3/2} X_M^{\nu}$ with $\nu \geq 0.5$ depending on what fraction of the stars are lost. $X_{\rm rlx}$ could be as large as ~ 20, indicating that the segregation-driven core collapse time could be much shorter in the embedded phase than estimated from observations

of young, gas-free clusters. Incidentally, the collision time increases by $X_{\rm coll} \approx X_R^{5/2} X_M^{\nu'}$ with $\nu' \leq 0.5$, a factor which can reach hundreds.

Such estimates suggest that the focus of the study of collisions in young clusters should be shifted to the embedded phase and to stars that are still in the process of formation, rather than MS objects. In this context, another process can lead to a strong increase of the stellar density, possibly up to a collisional phase (Bonnell, Bate, & Zinnecker 1998; Bonnell & Bate 2002). The orbits of stars accreting from the interstellar gas shrink as a consequence of conservation of linear momentum. If the gas has zero net momentum, does not dominate the potential, and is accreted on a timescale long compared to orbital time, the size of the stellar system will contract like $R_{\rm sys} \propto M_{\rm sys}^{-3}$, where $M_{\rm sys}$ is the combined, increasing mass of the stars. The above assumptions should be valid for the core of a forming cluster or sub-cluster, rather than the whole cluster, $N_{\rm sys} < N$. The overall contraction of the system is eventually halted by small-N stellar dynamical effects, in particular energy release by hard binaries. The maximum stellar density that can be reached is estimated to be $\rho_{\rm max} \equiv \langle m \rangle n_{\rm max} \approx \left(M_{\rm tot}/\langle m \rangle\right)^2 \bar{\rho}$, where $\langle m \rangle$ is the mean stellar mass reached at that time in the contracting system, $M_{\rm tot}$ the, total mass of the parent proto-cluster and $\bar{\rho}$ its average mass density. This density appears to be high enough for collisions to affect a significant fraction of stars only in rather rich systems, $N_{\rm sys} \gg 100$ (Clarke & Bonnell, in preparation).

Clearly the role of collisions in massive star formation or, more generally, in the dynamisc of young clusters can not be assessed independently of all other processes taking place in these complex systems. In this contribution, I have surveyed a few aspects that require consideration (see, e.g., Bally & Zinnecker 2005; Zinnecker & Yorke 2007 for other discussions of this subject). It seems that a complete picture will only be reached by means of numerical simulations combining stellar dynamics, the physics of the interstellar gas and the formation and early evolution of single and binary stars.

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